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The Interaction of Figurative and Operative Thinking, and Linguistic Barriers, in Elementary School Students' Graph-Based Numeracy Literacy: A Theoretical and Empirical Misconception Analysis

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ABSTRACT

Graph-based numeracy literacy is a core competence of the 21st-century primary curriculum. Yet student failure in this domain is typically attributed either to cognitive constraints or to linguistic difficulties, seldom to their interaction. This study examines elementary school students' misconceptions in solving graph-based word problems, focusing on how the interplay between figurative and operative thinking and linguistic barriers shapes numeracy literacy. The study employed an explanatory sequential mixed-methods design. Year 5 students completed a written test, and the results guided the purposive selection of a subgroup for subsequent think-aloud and stimulated recall interviews. Findings show that students' reasoning remains dominated by figurative thinking: they compare graph components additively and in isolation rather than integrating them into a coherent quantitative structure, so their answers reflect simple addition rather than substantive mathematical reasoning. Students also show limited command

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of relational terms such as "total" and "less than half," which distorts their choice of arithmetic operations. These findings suggest that the transition from figurative to operative thinking is a principal obstacle to numeracy literacy, and that addressing it requires a dual instructional strategy that combines varied problem-solving practice with explicit attention to mathematical language.

Keywords: Figurative thinking, graph-based numeracy literacy, linguistic misconception, numeracy error analysis, operative thinking

INTRODUCTION

In the 21st century, numeracy literacy involving data interpretation and statistical representation has become a crucial competence (Harsh et al., 2019; Hejnová, 2024). A key component of this literacy is the ability to solve graph-based word problems, which depends on students' capacity to convert textual narratives into valid quantitative models. Nevertheless, a range of studies show that this type of question remains a complex problem for both students and teachers (Moore et al., 2014; Stump, 2017). In brief, a substantial gap exists between the level of abstraction in graphical data representation and real-world problem-solving ability.

The root cause of this complexity likely lies in cognitive barriers closely tied to children's developmental level. At the primary level, students generally master only partitioning strategies, in which they tend to consider data components separately and cannot yet integrate them into a single, coherent body of information. This limitation constrains children's cognitive activity and causes them to rely predominantly on simple additive thinking (Domingo, 2021; Ding et al., 2022). From a Piagetian perspective, this phenomenon involves two types of mental activity performed by students (Moore et al., 2022). Students cannot yet perform this kind of integrative reasoning because figurative thought still dominates their thinking. As a result, they find it difficult to use operative thinking to connect various data components simultaneously. This limitation in mental activity causes minimal use of operative thinking at the level of the individual student. The impact of the clash between figurative and operative modes of thinking on students' misconceptions in reading graphs ultimately becomes the most significant challenge faced by classroom teachers.

Beyond this cognitive barrier, linguistic problems also matter, yet researchers often do not examine them in depth as a cause of student failure in solving mathematical word problems. Students often develop misconceptions when interpreting mathematical terms. They understand information in a fragmented way rather than integrating the problem's full intent into a coherent understanding (Fuchs et al., 2020). Although previous research has highlighted the importance of academic language proficiency for mathematical reasoning

(Moschkovich, 2015), few studies have examined the interaction between students' core cognitive reasoning mechanisms and their linguistic comprehension, particularly in solving word problems based on graphical representations. This integrative understanding is essential for successful problem-solving because students' conclusions depend not only on what they see visually but also on what they read textually.

In light of the gap identified in the literature above, this study was designed to systematically investigate Year 5 students' misconceptions when solving graph-based numeracy problems. This approach specifically elaborates on the respective roles of figurative and operative thinking within an integrated framework linking the development of linguistic competence to students' numeracy misconceptions. This study makes a dual contribution to mathematics education research by providing empirical insight into the direct relationship between basic cognitive developmental stages and students' numeracy errors, and by formulating pedagogical directions for teaching in primary classrooms. This study therefore aims to: 1) map the profile and frequency of students' errors in solving graph-based numeracy problems using the Newman Error Analysis hierarchy, and 2) analyse in depth how the dominance of students' figurative thinking interacts with semantic-linguistic barriers in triggering these mathematical misconceptions.

LITERATURE REVIEW

Piagetian Theory: Figurative and Operative Thinking in Numeracy Literacy

Numeracy literacy involving data interpretation and reasoning about quantitative data is mediated by an individual's cognitive structures. Numeracy literacy involving data interpretation and reasoning about quantitative data is mediated by an individual's cognitive structures (Harsh et al., 2019). The Piagetian theoretical framework, which distinguishes between figurative and operative thinking (Moore et al., 2022), provides a foundation for elaborating this phenomenon. Figurative thinking, as reviewed within Piaget's genetic epistemology framework by Moore et al. (2024), is rooted in information processing dominated by direct sensory perception of an object's visual characteristics. Consequently, students at the concrete operational stage tend to rely on what they see, remember, or what is directly represented before them.

In mathematical problems, students who predominantly think figuratively make common errors, such as applying simple partitioning strategies or basic additive reasoning when the total is not the main focus, instead comparing individual components separately. In contrast, operative thinking facilitates logical coordination and flexible manipulation of abstract schemas. As Ding et al. (2022) explains in a study on the development of multiplicative reasoning, students coordinate the part-whole relationship. They explicitly demonstrated that, without flexible

mental operations (operative thinking), students become trapped in simple additive reasoning - merely adding together the visible numbers - which is a characteristic of figurative thinking. Domingo (2021), in a study on students' cognitive behaviour in mathematical problem-solving, supports this view, explaining that students' ability to manipulate information abstractly, free from the visual trap of the object, depends heavily on the maturity of their operative cognitive functions. Although the concrete operational stage is transitional, internalisation of this phase often does not proceed optimally at the individual student level. As Toluk-Uçar and Bozkuş (2018) explain, many students who are chronologically at the concrete-to-formal operational stage still rely heavily on visual and concrete representations. When faced with complex data or pattern tasks, students frequently fail to make abstract generalisations and become trapped by what is physically visible in the problem.

A purely cognitive-developmental account, however, leaves an important question unanswered: through which observable stage of the problem-solving process does this reliance on figurative thinking actually surface? Newman's Error Analysis offers an empirical way to answer this by decomposing written mathematical problem-solving into a sequence of stages, allowing researchers to pinpoint such a cognitive limitation rather than merely infer it.

Newman Error Analysis

To trace how students' cognitive and procedural limitations are reflected in their written work, one diagnostic tool is Newman's Error Analysis (NEA) (Newman, 1977). Based on this framework, the process of solving written mathematical problems is evaluated through five hierarchical stages of error, namely:

1. Reading error, which occurs when students demonstrate an inability to read sentences, numbers, units, notation, words, or mathematical symbols accurately within a word problem;
2. Comprehension error, which occurs when students fail to understand and articulate the facts, whether certain or uncertain, presented in the problem. Students may record the data given in the problem but are unable, or only partially able, to state what is known and what is being asked, because they fail to grasp the meaning of that information;
3. Transformation error, which occurs when students fail to formulate a variable model, fail to plan a solution, or make an error in determining the arithmetic operation or formula that should be used to solve the problem;
4. Process skill error, which occurs because students fail to write down or are unable to perform calculations, carry out arithmetic algorithms incorrectly, or fail to reduce the calculation to its simplest form;
5. Encoding error, which occurs when students are unable to write the final answer completely and correctly, or neglect to state the appropriate unit at the end of the solution.

Applying this NEA hierarchy helps precisely identify the stage at which students' problem-solving procedures fail (Newman, 1977), including during the process of transforming visual data representations.

Two of these five stages-comprehension and transformation - are not purely computational: both require the student first to construct an accurate semantic representation of the problem text before any cognitive operation on the data can proceed. Newman's taxonomy identifies these stages but does not, on its own, explain why comprehension or transformation fails; that explanation requires attention to the problem's linguistic dimension.

Analysis of Linguistic Misconceptions in Mathematics

Students' internal cognitive barriers are often compounded by their external linguistic limitations. As Fuchs et al. (2020) explains, the main obstacle for students in solving word problems lies not merely in their computational ability, but in understanding the problem of text itself. Students must convert verbal narratives into mathematical models. This failure occurs when students cannot integrate linguistic (semantic) information with the required mathematical structure. If students fail to do so, the mathematical problem-solving process will break down operationally.

Moschkovich (2015) further explains that developing academic mathematical language proficiency is a fundamental component in facilitating higher-level reasoning. This condition is further complicated by the characteristics of graph-based assessment instruments, which demand a range of abilities such as reading the problem text, interpreting diagrams, and processing numbers and symbols. Students' failure to integrate these domains can then trigger misconceptions in mathematical reasoning.

Read together, Fuchs et al. (2020) and Moschkovich (2015) point to the same underlying requirement from two different angles: the former shows that comprehension failure already occurs at the level of the problem text, before any calculation begins, while the latter shows that this failure is compounded once the register shifts to academic mathematical language specifically. Neither account, however, addresses how this linguistic barrier interacts with the figurative-operative distinction discussed above, which the present study addresses.

Interaction of Cognitive-Linguistic Barriers in Graph-Based Numeracy Tasks

Four adjacent bodies of work bear the present problem, yet none of them address it directly. Studies of numeracy errors more broadly (Rufiana et al., 2024; Stump, 2017) document that graph-based word problems are hard for students, without asking why in cognitive or linguistic terms. Newman's Error Analysis supplies a stage-by-stage account of where an error occurs (Newman, 1977) but as noted above, is silent on why comprehension or transformation fails at a given stage. Research on Piagetian

cognitive development (Ding et al., 2022; Domingo, 2021; Toluk-Uçar and Bozkuş, 2018) explains the figurative-operative transition itself, but treats it as a purely cognitive phenomenon, independent of the language in which a problem is posed. And work on language-related difficulties in mathematical word problems (Fuchs et al., 2020; Moschkovich, 2015) explains how comprehension can fail linguistically, independent of a student's developmental stage. Each strand, in other words, holds one variable constant while examining the other. The present study departs from all four by treating the figurative-operative transition and linguistic comprehension as jointly responsible for the same error, rather than as alternative, competing explanations. Theoretically, the dominance of figurative thinking, which limits a student's ability to coordinate numerical data, is expected to exacerbate the impact of weak linguistic understanding of key mathematical terms, and vice versa. This reciprocal relationship forms the analytical framework this study refers to as the cognitive-linguistic dual constraint, and it is through this framework that the factors hindering the development of numeracy literacy among primary school students can be more fully explained.

METHODS

Research Design

This study employed an explanatory sequential mixed-methods design (Creswell & Plano Clark, 2018) to comprehensively examine the cognitive and linguistic aspects underlying students' misconceptions in solving graph-based numeracy problems. This design consists of two sequential stages: a quantitative stage, carried out first to identify the type and frequency of student errors, followed by a qualitative stage to explain and deepen these quantitative findings. The results of the first stage formed the basis for selecting participants for the second stage, allowing researchers to trace students' reasoning processes in greater depth and identify the relationship between figurative-operative thinking and linguistic errors.

Participants

The participants in this study were Year 5 primary school students aged 10-11 years. A total of 30 students completed the written diagnostic test at the initial stage of the study. Responses were first classified according to the five Newman's Error Analysis categories, generating an individual error profile for each student across the four test items. We then used purposive sampling based on two criteria. First, pattern consistency: a student was eligible for selection only if the same error category recurred in at least three items, indicating a stable rather than incidental misconception. Second, categorical representativeness: selection was distributed to cover both dominant error categories identified in the quantitative screening—transformation and comprehension—along with at least one less frequent category so that the subsequent qualitative phase would capture the diversity of misconception types rather than only the most common

pattern. Applying these criteria to the 30 error profiles yielded four eligible students; their think-aloud and stimulated-recall data are presented in the Results section. Year 5 was selected because students at this age are at the concrete operational stage, so the transition from figurative to operative thinking was expected to be clearly reflected in the learning difficulties they experienced. This study obtained ethical approval from the relevant institution, as well as written consent from the school principal and the students' parents.

Research Instruments

This study used two main data-collection instruments, both validated through expert judgment before the research. Two validators carried out the validation: a lecturer in mathematics education and a senior mathematics teacher with more than ten years of classroom experience. Both validators reviewed each item for content relevance, wording, and grade-level appropriateness. Their feedback led to revisions focused mainly on sentence structure; several items were rewarded to match the reading level and vocabulary of Year 5 primary school students while preserving the mathematical content being tested. The first instrument was a written test consisting of graph-based word problems with narrative contexts. These items were designed to assess students' ability to interpret quantitative data, integrate information from several graph components, and apply simple additive and multiplicative reasoning. The items were specifically designed to elicit data-component separation errors that reflect students' operative thinking.

The second instrument was a semi-structured interview guide, reviewed by the same two validators, used to trace the cognitive processes underlying students' written answers. This interview employed two techniques: think-aloud, which asked students to verbalise their thinking steps as they solved the problem, and stimulated recall, which asked students to explain the reasoning behind their previously incorrect written answers.

Data Collection Procedure

Data collection was carried out over four weeks in three stages. In the first stage, the previously validated written test was administered to 30 students under standardised classroom conditions and with adequate time allocation. In the second stage, students' answers were analysed to identify the types of errors that emerged and categorised according to Newman's Error Analysis (NEA). Based on this identification, four students were purposively selected for interviews. In the third stage, I conducted individual interviews lasting 45 to 60 minutes with each student. I recorded the entire interview process using an audio recorder to capture students' thinking processes fully.

Data Analysis

Data analysis was conducted in two stages, in line with the explanatory sequential design. In the first (quantitative) stage, students' written test answers were statistically

Table 1
Frequency distribution of types of student error based on Newman's error analysis

Newman Error Stage	Frequency	Percentage (%)	Dominant Characteristic
Reading	2	6.67	Errors in reading basic graphical terms
Comprehension	9	30.00	Failure to interpret quantitative relational phrases
Transformation	13	43.33	Inability to convert data
Process Skills	4	13.33	Computational errors
Final Answer Writing	2	6.67	Mismatch between the conclusion and the context of the problem

Source: Authors' own work

classified using the Newman Error Analysis (NEA) framework into five error categories: reading, comprehension, transformation, process skills, and final answer writing. This classification showed which aspects of students' reasoning were most prone to particular error types and served as the basis for selecting participants for the interview stage. In the second (qualitative) stage, I analysed interview transcripts using semiotic and thematic analysis. A constant comparative method compared the written errors identified in the first stage with students' reasoning in the second stage. This comparison helped reveal emerging themes related to figurative-thinking dominance, limitations in additive logic, and language-comprehension barriers experienced by students. This comparison between the two stages was carried out to enhance the validity and trustworthiness of the research data.

RESULTS

Data analysis in this study was carried out in stages, beginning with a quantitative mapping of error types based on Newman's Error Analysis framework (Newman, 1977), followed by a deeper qualitative thematic analysis of the linguistic barriers underlying these errors (Fuchs et al., 2020).

Quantitative Mapping of Student Errors

Classification of the written answers of 30 students according to Newman's scheme shows that student errors were not evenly distributed but were concentrated at particular stages, as described in Table 1. This analysis determined error categories based on the first error encountered along the student's cognitive pathway according to the NEA hierarchy, so each instance of student error was classified exclusively at its earliest stage of failure.

Table 1 shows that most student errors occurred at the transformation stage (43.33%) and the comprehension stage (30%). This indicates that students' main barrier is not computational skills but coordinating visual data into a coherent mathematical model. Students' inability to integrate various quantitative data components is consistent with the characteristics of cognitive

development at the concrete operational stage, within Piaget's genetic epistemology perspective. These quantitative findings formed the basis for selecting four students for in-depth interviews to trace the four misconception patterns described below.

Thematic Analysis of Student Misconceptions

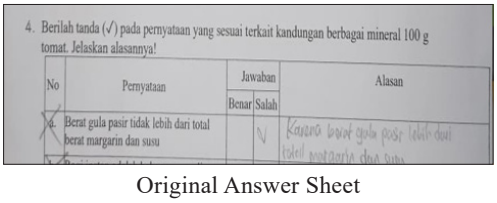
Separation or Pairwise Comparison Errors

This pattern was identified in item 4a, which tested students' ability to verify the relationship between the weight of sugar (100 grams) and the combined weight of margarine (75 grams) and milk (30 grams). Mathematically, the statement "the weight of the sugar is no more than the combined weight of the margarine and milk" is true, since 100 grams does not exceed 105 grams (75+30). S1 and S2 both marked this statement as false.

When traced through interviews, Figure 1 explained that he compared the sugar with the margarine, and then the sugar with the milk, separately, without adding the two together first. Even when asked directly whether he had tried to add the numbers together, S1 confirmed that he had only compared them one by one.

A similar pattern emerged for Figure 2, who described his solution strategy as comparing sugar with margarine, then sugar with milk, without any integration stage.

Both cases show that students process data based on partial visual perception of the components. This reflects the dominance of figurative thinking within Piaget's framework, which emphasises reliance on direct perception of concrete objects rather than abstract mental transformation.



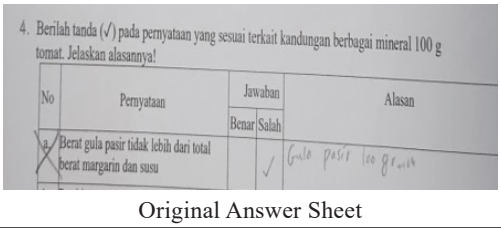
Original Answer Sheet

Mart (v) the statement that is correct regarding the mineral content of 100 grams of tomatoes. Explain your reasoning!

No	Question	Answer		Reason
		Correct	Incorrect	
1	The weight of granulated sugar is not more than the total weight of margarine and milk.		v	Because the weight of granulated sugar is more than the total weight of margarine and milk

Translation

Figure 1. S1's written answer
Source: Student answer scripts



Original Answer Sheet

Mart (v) the statement that is correct regarding the mineral content of 100 grams of tomatoes. Explain your reasoning!

No	Question	Answer		Reason
		Correct	Incorrect	
1	The weight of granulated sugar should not exceed the total weight of margarine and milk.		v	100 grams of granulated sugar

Translation

Figure 2. S2's written answer
Source: Student answer scripts

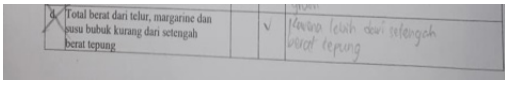
Failure to Integrate Multiple Quantities Simultaneously

This error appeared in students' work on item 4d, which tested whether the combined weight of eggs (125 grams), margarine (75 grams), and powdered milk (30 grams) was less than half the weight of flour (500 grams). Mathematically, this statement is true, since 230 grams (125+75+30) is less than 250 grams (half 500 grams).

Figure 3 shows that S3 successfully added the three components correctly (125+75+30=230) yet still marked the statement as false. When the researcher asked what was meant by "half the weight of the flour", S3 merely repeated the phrase "half the weight of the flour" without being able to state its value and fell silent when pressed further. This pattern indicates that, although S3's basic arithmetic skills were adequate, he was not yet able to coordinate the result of this addition with the second comparative concept ("half") simultaneously. This indicates a limitation in operative reasoning typical of students at the concrete operational stage.

Misconceptions Regarding Academic Mathematical Language

An evaluation of answer sheets across all subjects (S1 to S4, as documented in Figures 1 to 4) revealed a recurring pattern of misunderstanding relational terms such as "total", "no more than", and "half of". Students could not interpret these terms as part of a coherent quantitative relationship and instead used keyword-based computational strategies. This failure to form a coherent semantic representation from the verbal context supports the argument that academic language distortion is a major determinant of mathematical error, even when students' numerical computation ability is adequate.

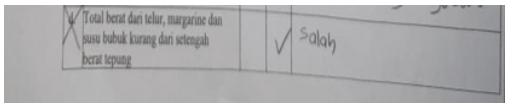


Original Answer Sheet

No	Question	Answer		Reason
		Correct	Incorrect	
1	The total weight of the eggs, margarine, and powdered milk is less than half the weight of the flour		v	because it is more than half the weight of the flour.

Translation

Figure 3. S3's written answer
Source: Student answer scripts



Original Answer Sheet

No	Question	Answer		Reason
		Correct	Incorrect	
1	The total weight of the eggs, margarine, and powdered milk is less than half the weight of the flour		v	incorrect

Translation

Figure 4. S4's written answer
Source: Student answer scripts

Confusion between Comparison, Equality, and Inequality

This pattern is clearest in subject S3. When asked what item 4d was actually asking, S3 initially understood it as a question about equality ("is the combined weight of the eggs, margarine, and powdered milk equal to half the weight of the flour") and only corrected this to "less than" after being asked to re-read the question slowly.

Although S4 (in Figure 4) correctly calculated the total weight (230 grams) and knew it was less than 250 grams, he still marked the statement as false. This occurred because he interpreted the statement as requiring equality between the two quantities rather than recognising that "less than" denotes an inequality. This finding reveals a subtle misconception than the previous themes: for S4, the issue lay not in computational ability but in understanding the statement's relational meaning.

Difficulty in Transitioning from Figurative to Operative Thinking.

A synthesis of the four clusters of findings above confirms one fundamental characteristic: students at the concrete operational stage show a strong dependence on data-separation strategies based on direct sensory-visual impressions, and have not yet achieved the mental-transformation capacity required to coordinate quantitative components simultaneously. Limited mastery of linguistic structure further accelerates this barrier to operative reasoning. Thus, the errors identified in this study result from a reciprocal interaction between cognitive-capacity constraints and linguistic-competence limitations

DISCUSSION

The Dominance of Figurative Thinking over Operative Reasoning

The findings confirm that Year 5 primary school students predominantly adopt a separation strategy when solving graph-based word problems (Moore et al., 2022; Liang & Moore, 2021). This reasoning difficulty aligns with Manmai et al. (2021), who found that among primary school students, the cognitive aspects of mathematical reasoning habits tend to remain limited to skills and procedures and are difficult to develop directly without tasks specifically designed to promote deeper reasoning. This phenomenon shows that, even though students are chronologically at the concrete operational stage, figurative reasoning still intervenes in their thought processes. As a result, students experience fundamental difficulty in constructing coherent quantitative explanations from the various data sources presented (Moore et al., 2022). This difficulty aligns with Friel et al. (2001), who argue that graph comprehension is not merely the ability to read data literally, but requires connecting various graphical elements into a single, coherent relational structure.

Although Piaget's theory of cognitive development assumes a linear and consistent transition toward operative logic, the empirical data in this study show that subjects have not yet fully constructed additive reasoning within their thinking

schemas. This barrier to conceptual data integration is consistent with the findings of Domingo (2021) and Ding et al. (2022), who identified primary school students' inability to reconstruct abstract relational structures, such as understanding the comparative significance of terms such as “greater than” or “no more than”.

The tendency to compare data components separately, without overall aggregation, indicates a strong attachment to sensory-visual representation. Figurative thinking limits students' cognitive flexibility because they conclude exclusively from the physical impressions of concrete objects directly observed by the senses, rather than through abstract mental transformation (Moore et al., 2022; Liang & Moore, 2021). This pattern aligns with the concept of “graph sense” proposed by Friel et al. (2001), whereby a holistic sensitivity to the meaning of graphs develops gradually and is not necessarily present in primary school students.

This transitional condition at the concrete operational stage leads students to fail to identify the structural relationship between the parts and the whole of the data (Domingo, 2021). Students' reasoning remains isolated at the level of partial addition operations, without being able to convert this into a substantive operative conclusion. This immaturity of a flexible and reversible operative logic system explains why the basic computational procedures students master fail to lead them to a valid mathematical solution (Ding et al., 2022).

Interaction between Cognitive Barriers and Linguistic Constraints

Barriers in mastering academic mathematical language further exacerbate the cognitive factors described above. Students' inability to conceptualise relational phrases such as “total weight” or “less than half” forces them to shift their focus onto textual characteristics. This phenomenon validates the arguments of Fuchs et al. (2020) and Prediger et al. (2018) that students' mental activity and mathematical reasoning are strictly moderated by their linguistic competence. This aligns with the meta-analytic review by Vessonen et al. (2024), which argues that cognitive load and linguistic characteristics in mathematical word problems are not independent factors but interact significantly. Problems that contain lexical inconsistencies while also requiring the integration of several solution steps are the most difficult for primary school students. Students' inability to conceptualise relational phrases such as “total weight” or “less than half” forces them to focus solely on textual characteristics rather than the relational structure embedded within them.

Students frequently fail to construct an integrative semantic and mathematical representation from the verbal context because they become trapped in keyword-based computation. When faced with complex word-problem structures, students isolate particular literal keywords (for example, extracting the word “half”) without connecting them to the overall quantitative relationship. This linguistic limitation confirms the crucial role of academic

mathematical language as a semantic bridge between the visual representation of numerical data and the conceptual aspects of the expected mathematical model (Moschkovich, 2015). Failure to understand the direction of textual instructions breaks the chain of quantitative-information coordination, demonstrating that language competence and contextual understanding are integral determinants of students' achievement of numeracy literacy.

Pedagogical Implications: A Dual Instructional Strategy

Pedagogically, the convergence of these cognitive and linguistic barriers underscores the need to implement a dual instructional strategy in primary school mathematics teaching. This urgency is reinforced by the findings of Moosapoor (2023), who reported that novice primary school teachers still have limited ability to specifically recognise the types of mathematical misconceptions students experience, meaning that instructional interventions designed based on actual student error patterns - such as those identified in this study - are crucial in filling this gap.

One instructional intervention could strengthen operative reasoning. Instructional design should focus on training students to transition from figurative to operative thinking through modelling tasks that require integrating multiple quantitative parameters, rather than drill-based exercises. Another intervention involves developing linguistic competence. This can be achieved by restructuring problem-based exercises to connect direct verbal expressions with arithmetic operations. Modification activities such as paraphrasing tasks for word-problem texts, visual modelling, and explicit instruction on academic relational terminology can reduce students' visual dependence on figurative thinking.

The synergy between strengthening additive logic and mathematical language literacy is a key tool that enables students to overcome procedural barriers, construct coherent problem representations, and solve numeracy problems of higher cognitive complexity.

CONCLUSION

This study examines elementary school students' misconceptions in solving graph-based numeracy problems by relating the stages of Newman Error Analysis to the interaction between figurative and operative thinking, as well as students' understanding of mathematical language. Quantitative analysis shows that errors concentrate in the transformation stage (43.33%) and the comprehension stage (30.00%), indicating that students' difficulties extend beyond computational procedures to the construction and coordination of mathematical meaning. Further qualitative findings show that students often process graphical information as separate components rather than coordinating these quantities into a coherent relational structure. In some cases, students could perform the necessary arithmetic operations but could not use the calculated quantities to evaluate the relationships stated in the problem.

Qualitative evidence also suggests that linguistic understanding plays a significant role in these difficulties. Terms and expressions such as "total," "no more than," and "half of" are not consistently interpreted as relational mathematical statements. Some students rely solely on specific keywords or interpret comparative statements as requiring equal values, even when their calculations yield values that could actually support the correct conclusion. These patterns suggest that computational performance alone cannot fully explain errors in graph-based numeracy tasks. Rather, these findings highlight the interaction between students' limitations in coordinating quantitative information and their difficulties in constructing accurate semantic representations of the problem. Within the Piagetian framework adopted in this study, this interaction is reflected in the continued dominance of figurative processing and the incomplete development of operative coordination. Therefore, these findings position the transition from component-by-component visual processing toward coordinated operative reasoning as a critical issue in the development of graph-based numeracy literacy among elementary school students.

IMPLICATION OF THE STUDY

These findings have implications for designing numeracy instruction, particularly when students are asked to interpret graphical information in word problems. Instruction should not treat the interpretation of graphs, mathematical language, and computation as separate competencies. The errors identified in this study indicate that students may possess sufficient procedural knowledge to perform arithmetic operations but fail to establish the quantitative relationships required by the problems. Consequently, instructional activities should give students opportunities to integrate multiple pieces of information before selecting an operation or forming a conclusion. Tasks that require students to compare quantities, combine multiple data components, and explain the relationship between parts and the whole can provide a more appropriate context for developing operational thinking than exercises that focus primarily on obtaining numerical answers.

These findings also have implications for the teaching of mathematical language in elementary school classrooms. Expressions such as "total," "no more than," and "half of" should be understood as mathematical relationships, not merely as isolated vocabulary terms associated with specific operations. Teachers can ask students to restate on a problem in their own words, represent these relationships with numbers or visual models, and explain why a particular comparison is valid. These activities can help build semantic connections between a problem's wording and its underlying quantitative structure. This is particularly relevant for graph-based tasks, where students must integrate information presented through both textual and visual representations.

Regarding assessment and learning diagnosis, study results indicate that

simply determining whether an answer is correct or incorrect does not provide sufficient information about the source of a student's difficulties. The use of Newman's Error Analysis can help distinguish between difficulties in understanding and transformation from errors that occur during calculation or the formulation of the final answer. Combining this diagnostic information with students' explanations of their reasoning can provide a more accurate basis for teachers to determine whether the errors stem from a misunderstanding of mathematical language, a failure to coordinate graphical information, or difficulty in carrying out the necessary procedures. This diagnostic differentiation can guide further learning support, rather than treating all incorrect answers the same.

LIMITATIONS AND RECOMMENDATION FOR FUTURE RESEARCH

This study has several methodological limitations that should be considered when interpreting its results. First, the research scope was limited to an in-depth qualitative analysis of four representative subjects (S1 to S4) at Year 5 primary school level. Although this approach provides an in-depth understanding of the cognitive and linguistic mechanisms underlying student errors, the limited sample size does not support broad statistical generalisation regarding the prevalence of figurative reasoning across the wider population of students at the concrete operational stage. Second, the assessment instrument focused on graph-based word problems in the specific context of ingredient quantities and therefore did not examine students' numeracy literacy performance on other types of visual data presentation with different levels of complexity, such as pie charts or multi-trend line graphs. Finally, the linguistic analysis identified only manifestations of misconceptions regarding certain academic mathematical terminology. Still, it did not measure the contribution of students' general reading comprehension ability to mathematical reasoning distortion in isolation.

Given these limitations, several directions may be pursued for future research. First, follow-up studies could employ a mixed-methods design involving a larger random sample to provide a more comprehensive picture of the distribution of partitioning strategies and the development of additive reasoning among elementary school students. Second, experimental studies or classroom action research are needed to empirically test the effectiveness of the proposed dual instructional strategy—which combines mathematical modelling with academic language reinforcement—particularly in reducing students' reliance on figurative reasoning. Third, future research should include standardised reading literacy assessments to distinguish general linguistic barriers from difficulties specifically related to the semantic processing of academic mathematical language. Finally, comparative studies involving various forms of graphical representations—including single-bar diagrams, pie charts, and complex infographics—could examine how

variations in visual formats affect students' ability to transform visual information into valid operational mathematical models.

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